Computer assisted theorem proving for the education

or

Computer assisted education for theorem proving?

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The motivation

The difference

- Computer assisted theorem proving for the education

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goals of mathematics education are a priori settled and ATP-DG is considered as an artifact to (help) achieve some given goals.

- Computer assisted education for theorem proving

===>

goals of mathematics education are to be reconsidered in view of the popularization of DG and of ATP in DG.

Questions

- Is Automated Theorem Proving in geometry education good for anything?
- If yes, what is Automated Theorem Proving in geometry education good for?
- What should be the necessary changes and requirements in the educational context, if ATP is to be considered good for something....?

Long time ago...

- ICMI Study: “School Mathematics in the 1990's” (Kuwait, 1986)

“even if the students will not have to deal with computers till they leave school, it will be necessary to rethink the curriculum, because of the changes in interests that computer have brought.
Let us mention here just three of them:
a) Algorithms, b) Discrete mathematics, c) Symbol manipulation.”

“it seems appropriate, in the last (secondary school) grades, to bring attention to issues such as algorithmic efficiency, and to distinguish between those, say, with polynomial and with not polynomial complexities"

Consider, for example, the following question (to other aspects of which we shall wish to refer later):

Two lines are drawn from one vertex of a square to the midpoints of the two non-adjacent sides. They divide the diagonal into three segments (see Figure 5.2).

(a) Are those three segments equal?

(b) Suggest several ways in which the problem can be generalised.

(c) Does your answer to (a) generalise?

(d) Can the argument you used in (a) be used in the more general cases?

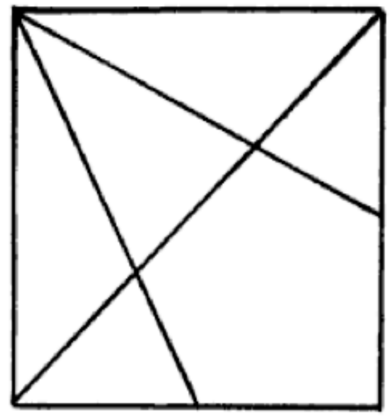


Fig. 5.2

(e) If your answer to (d) is 'No', can you find an argument which does generalise?

We shall comment briefly on the purpose of such questions in Chapter 6. Here we note only that whereas the student of the 1950s had only purely geometrical ways of tackling the problem, today's student may well be able to apply algebraic methods to find the solution for (a). The solution derived by applying a mechanical procedure may be less aesthetically satisfying than a geometrical one, but are there other objections to algebraic methods than that of aesthetics? Arguments against the use of algebraic methods for the solution of geometric problems have a long history: Simson in the eighteenth century, we are told, regarded such methods as little better than a 'mechanical knack' in which the student proceeded 'without ideas of any kind' in order to 'obtain a result without meaning and without being conscious of any process of reasoning'. Similar objections were used by those supporting Euclid's geometry in the early nineteenth century, for example, Ohm in Germany and Whewell in England. They were more recently repeated by Thom (1973) who pointed out how algebra is rich in 'syntax', but weak in 'meaning', whereas geometry is the reverse.

“the solution obtained by the application of a mechanical procedure could be aesthetically less satisfactory than one of geometric style, but, are there objections other than aesthetical?”

ICME 96

- 1996 ICME 8-TG19, *Computer-based interactive learning environments*, N. Balacheff-J. Kaput-T.R. http://mathforum.org/mathed/seville/followup.html

"...A recent trend of research is to *link powerful tools such as theorem provers, to microworlds in order to support students exploration of mathematical properties*, testing of conjectures, and searching for counter-examples. Tomas Recio presented the use of the computer algebra software CoCoA to support the exploration of elementary Euclidean geometry theorems, suggesting that this program could be thought as the core of a future intelligent, interactive, learning environment linked to a sketch tool such as Cabri-Geometre. Philippe Bernat illustrated this trend in development of CBILEs, which consist of augmenting a microworld with "reasoning tools", with the project CHYPRE which aims to give freedom to explore a problem in any way and to test any plan of problem-solving.

The trend in design is clearly to develop environments specific to mathematics and provide means for students to express their ideas about objects and relations, and possibly their reasoning as well. Some participants expressed their worry that all these developments may be technology pushed, whereas the Panel argued on the contrary that they are user-&-mathematics driven. Mathematics is at the core of modern CBILEs, but the complexity of their contribution to learning is questionable to teachers considering their everyday practice. This issue has also been addressed in a pragmatic way...."

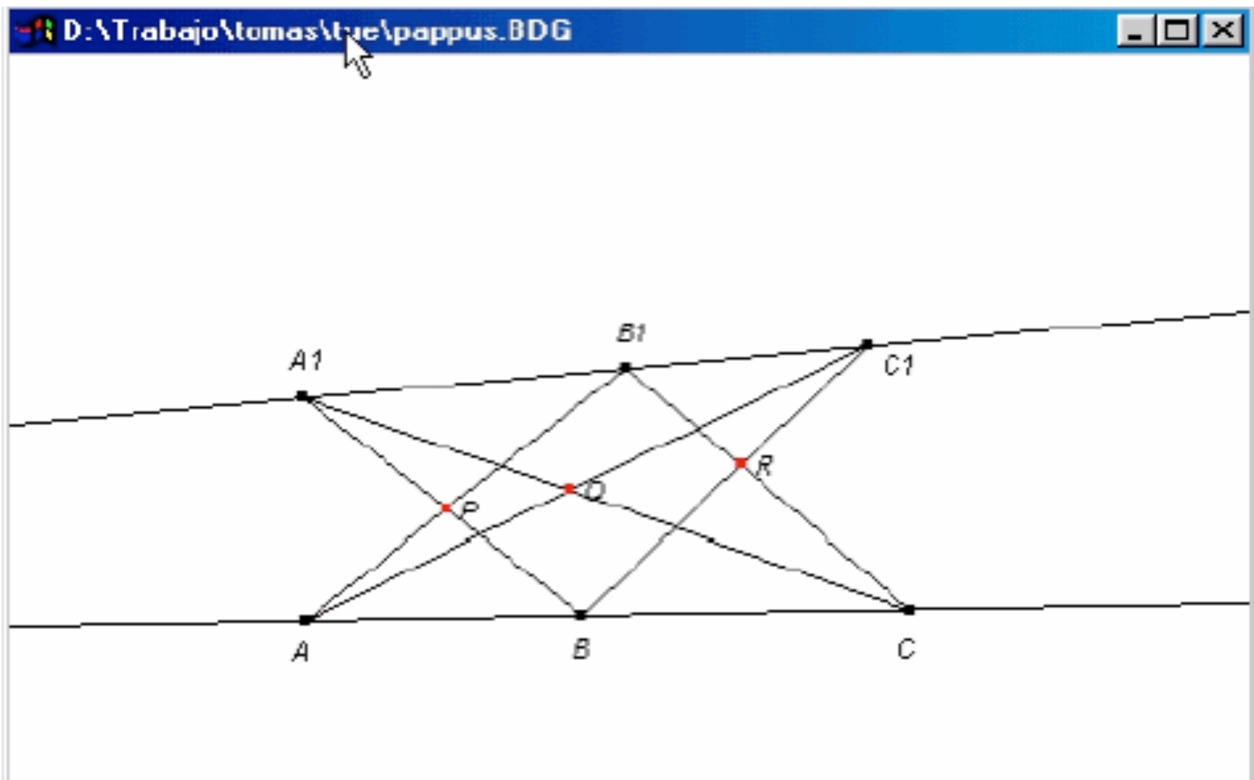
Goal

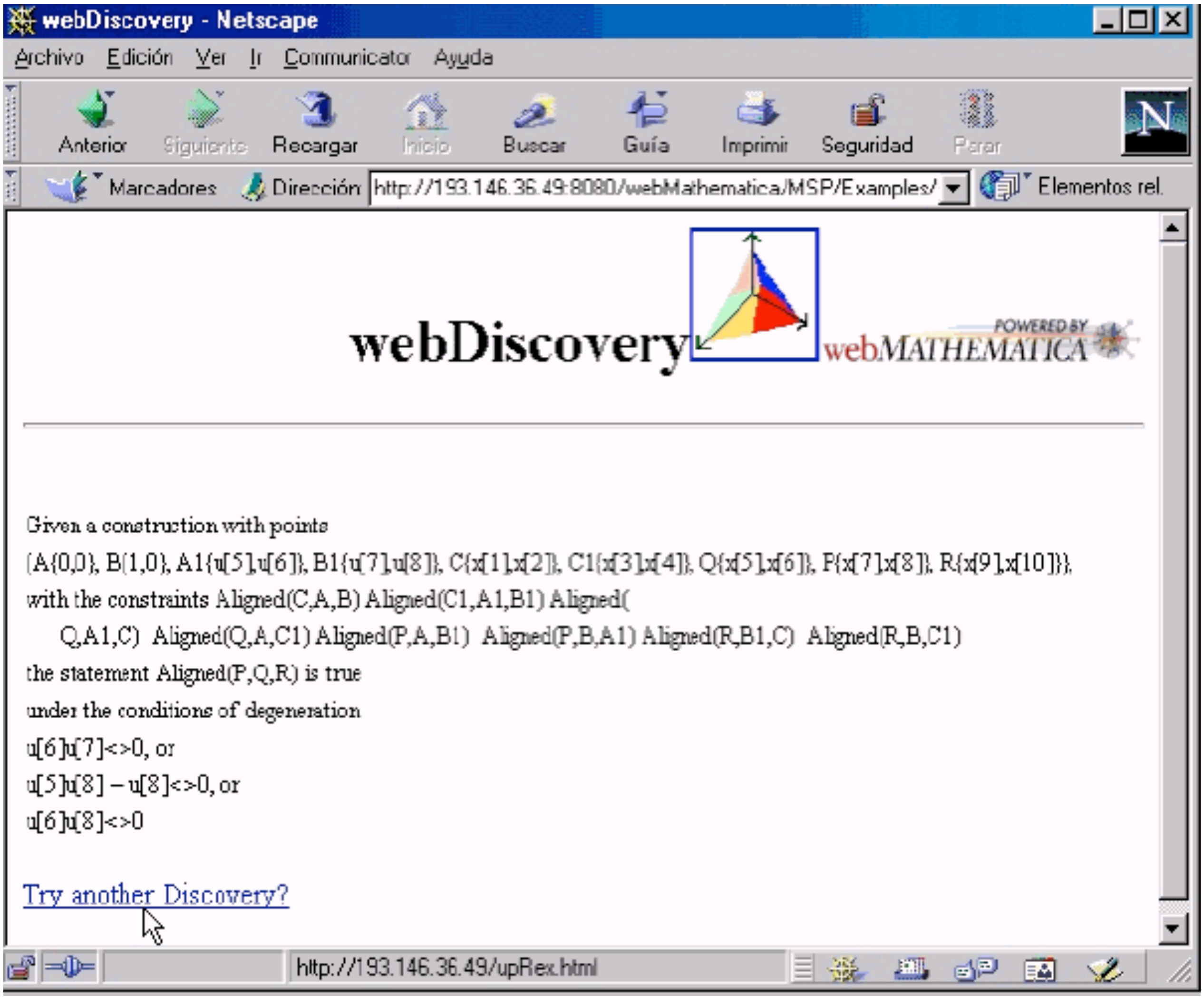
- Check if some statement is true or false

H ==> T?

GDI ~2000

F. Botana http://webs.uvigo.es/fbotana/
J.L. Valcarce

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Given a construction with points
{A{0,0}, B{1,0}, A1{u[5],u[6]}, B1{u[7],u[8]}, C{x[1],x[2]}, C1{x[3],x[4]}, Q{x[5],x[6]}, P{x[7],x[8]}, R{x[9],x[10]}},
with the constraints Aligned(C,A,B) Aligned(C1,A1,B1) Aligned(
Q,A1,C) Aligned(Q,A,C1) Aligned(P,A,B1) Aligned(P,B,A1) Aligned(R,B1,C) Aligned(R,B,C1)
the statement Aligned(P,Q,R) is true
under the conditions of degeneration
u[6]u[7]<>0, or
u[5]u[8] – u[8]<>0, or
u[6]u[8]<>0
Try another Discovery?
http://193.146.36.49/upRex.html

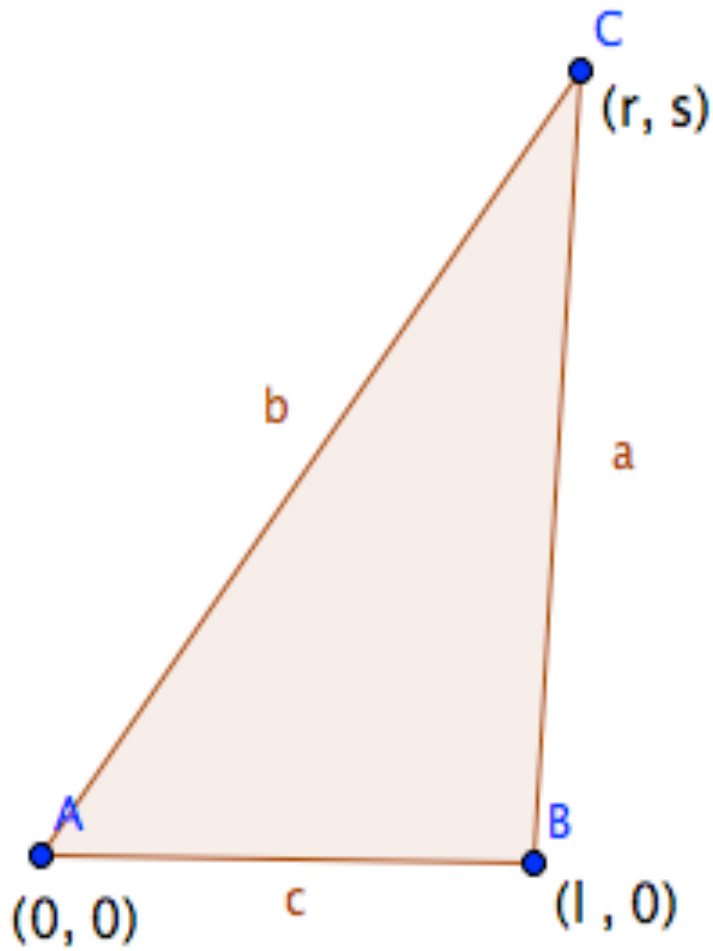
Points	LingProperties
A(0,0)	Aligned(C,A,B)
B(1,0)	Aligned(C1,A1,B1)
A1(u[5],u[6])	Aligned(Q,A1,C)
B1(u[7],u[8])	Aligned(Q,A,C1)
C(x[1],x[2])	Aligned(P,A,B1)
C1(x[3],x[4])	Aligned(P,B,A1)
Q(x[5],x[6])	Aligned(R,B1,C)
P(x[7],x[8])	Aligned(R,B,C1)
R(x[9],x[10])	

LingConditions
Aligned(P,Q,R)

ProveProperties

- Obtain all conclusions from a geometric diagram (or picture)

Example: Heron's formula




```
> restart: with(PolynomialIdeals):
> II:=<2*S-l*s, a^2-(r-l)^2-s^2, b^2-r^2-
  s^2, c^2-l^2>;
```

$$II := \langle a^2-(r-l)^2-s^2, c^2-l^2, -l\,s+2\,S, b^2-r^2-s^2\rangle \quad \textbf{(3.2.1.1)}$$

```
> EliminationIdeal(II,{S,a,b,c});
```

$$\langle a^4-2\,a^2\,b^2-2\,a^2\,c^2+b^4-2\,b^2\,c^2+c^4+16\,S^2\rangle \quad \textbf{(3.2.1.2)}$$

```
> FF:=<2*S-l*s, a^2-(r-l)^2-s^2, b^2-r^2-
  s^2, c^2-l^2,2*p-(a+b+c)>;
```

$$FF := \langle a^2-(r-l)^2-s^2, 2\,p-a-b-c, c^2-l^2, -l\,s+2\,S, b^2-r^2-s^2\rangle \quad \textbf{(3.2.1.3)}$$

```
> (S^2-p*(p-a)*(p-b)*(p-c)) in FF;
```

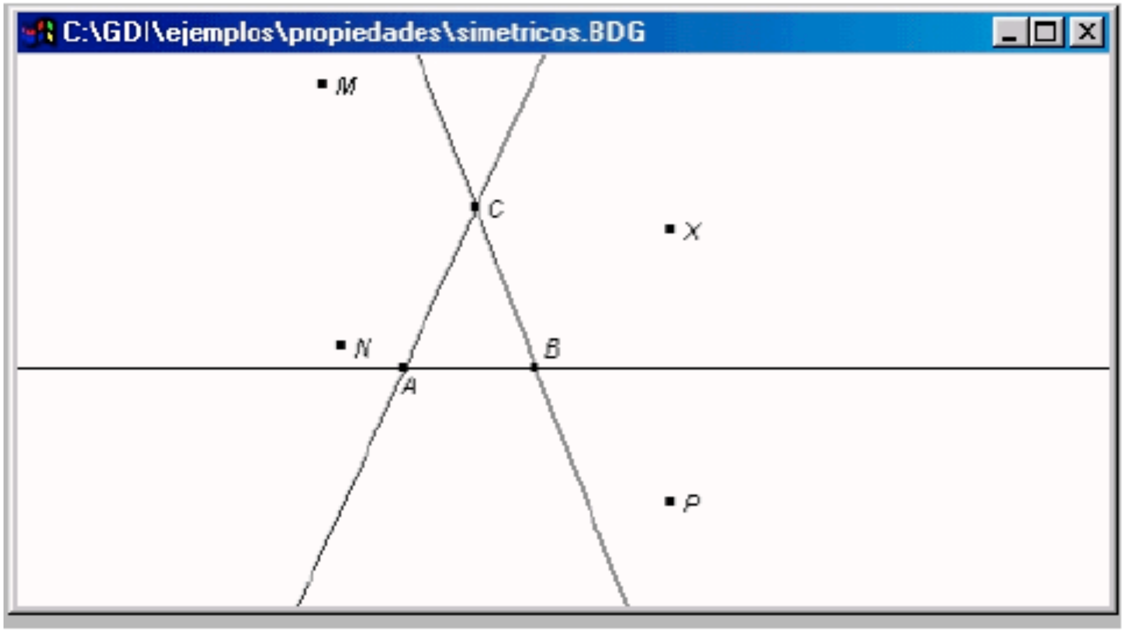
true (3.2.1.4)

Given H, find all T's such that H ==> T

• Find complementary hypothesis for the truth of a conjectured statement

simétricos

Given a triangle ABC and a point X, let M,N,P, be the symmetric image of X with respect to the sides of the triangle. Then M, N, P are aligned.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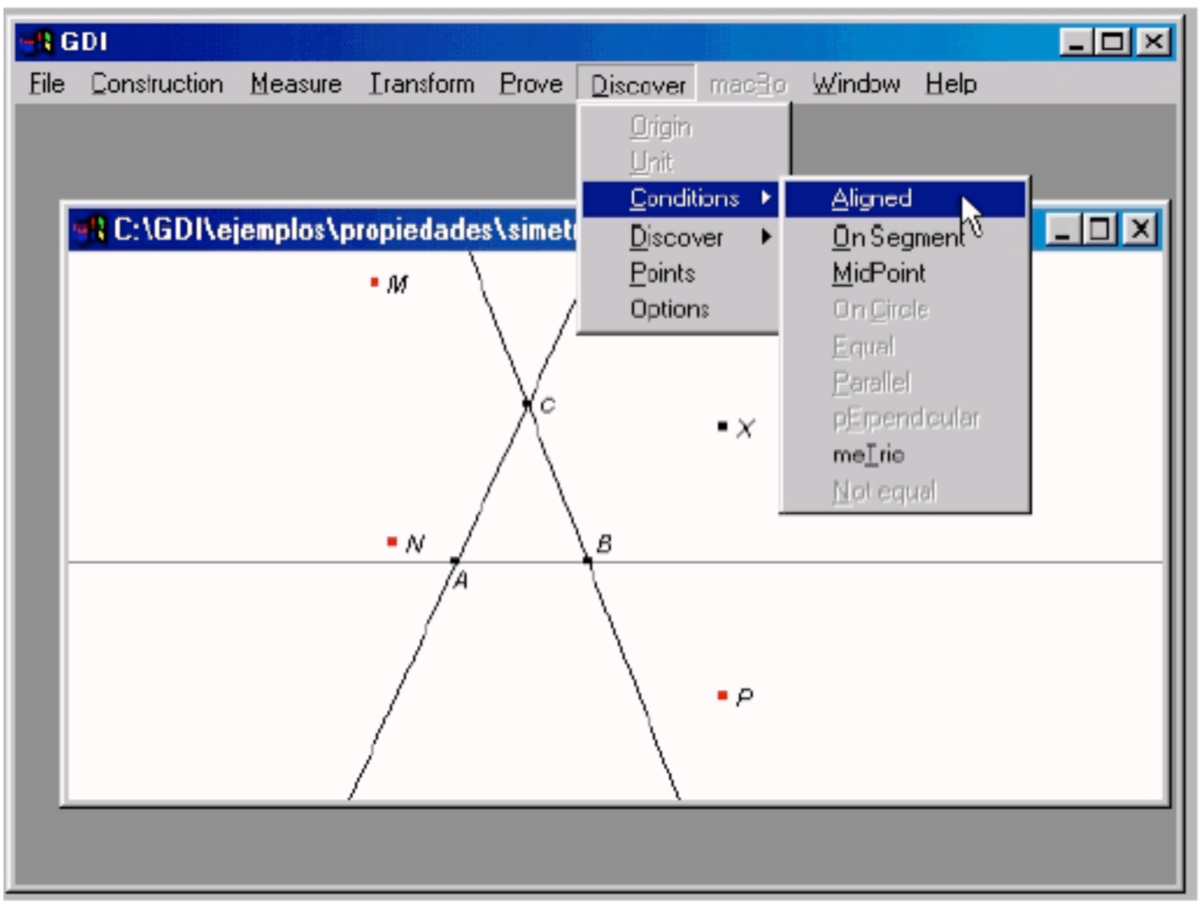
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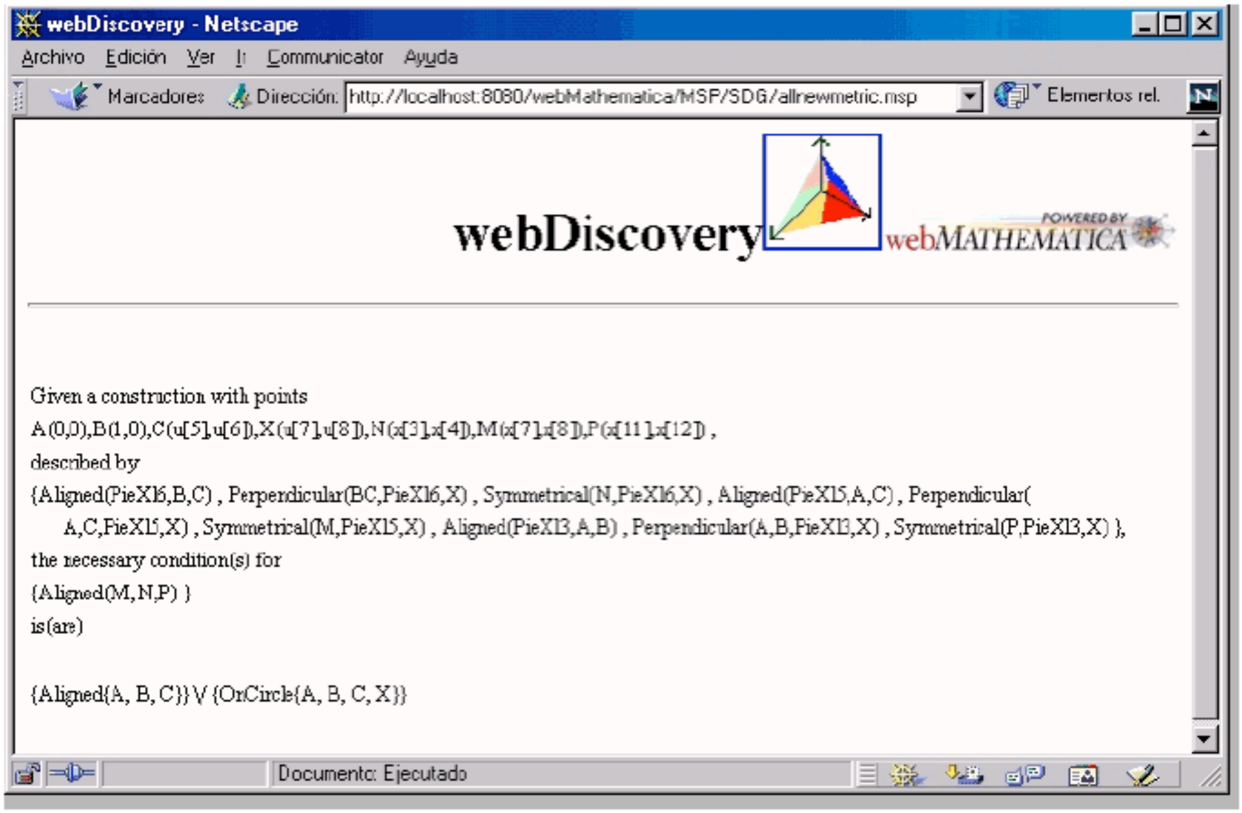

Given H and T, find H' such that H & H' => T

OMNISCENT.... as a teacher!

And yet...

• ATP-DG is considered to be helpful as an "omniscient" consultant working side by side with the student. It is NOT to replace what is usually called "geometric reasoning", in case this is a desirable goal..

P. Richard. *Raisonnement et stratégies de preuve dans l'enseignement des mathématiques.* Ed. Peter Lang. Berne. 2004.

AgentGeom
GeoGebra Tutor

https://www.researchgate.net/profile/Josep_Fortuny2/publications
https://www.researchgate.net/profile/Philippe_Richard7/publications/2

• But

Until now, the Dynamic Geometry conception is, in some sense, using new technology for old problems (old problems that are intimately "old tools"-driven)

• Will we be interested in case-dependent reasoning when we have a universal method?

• Mathematical knowledge is intimately bound to its setting: knowledge placed/knowledge learned

"...How, exactly, can we systematically specify the relationship between knowledge placed within a system by a designer, and knowledge constructed by a learner as she or he interacts with it?..."(Richard Noss)

• Reification (Verdinglichung) of mathematical knowledge in computer-based learning environments, and accompanying enrichment of mathematical experience due to progress in interface design and knowledge representation

• New technology enlarges the field of geometric objetcs subject to formal reasoning: for instance, simultaneous operations with many geometric objects.
Moreover, it changes the antropomorphic approach to geometry: objects are not "seen"!.... but "stored". Manipulation is, essentially, that of data structures. Euclidean, elementary geometry is required to reason about

them.

Davis

The Rise, Fall, and Possible Transfiguration of Triangle Geometry: A Mini-History, Philip J. Davis. The American Mathematical Monthly, Vol. 102, No. 3. (Mar., 1995), pp. 204-−214.

https://en.wikipedia.org/wiki/Philip_J._Davis

6. THE TRANSFIGURATION OF TRIANGLE GEOMETRY. Can a subject arise from the dust and ashes that history has piled on it? Only if it is transformed in the process. The focus of triangle geometry has now been changed. The computer has popped it up a metalevel, and in the process has transfigured the subject. Hundreds of elementary and not so elementary theorems that were in the literature have now been proved by computer. Many new theorems have been discovered, again in a variety of ways. Triangle geometry always was a practice ground for strategies of proof in the spirit of Euclid, and it has now become a testing ground for strategies of decidability, proof, and theorem discovery. These strategies have run from naive schemes to the employment of deep and abstract results of modern algebra and differential algebra.

*As regards mathematical education, I think the message is clear. Classical proof must move over and share the educational stage and time with other means of arriving at mathematical evidence and knowledge. Mathematical textbooks must modify the often deadening rigidity of the Euclidean model of exposition.

Kaput

Jim Kaput (92):

http://www.kaputcenter.umassd.edu

"..predicted

a continuing transition from Doing (old) Things Better to Doing Better Things.

Let us take this last sentence as a challenge for teachers and researchers for the coming decade" (Balacheff)

- We need to rethink not only **how** to teach but **what** to teach...